# On-line Construction of Compact Suffix Vectors and Maximal Repeats

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# Plan

1. Introduction
2. Suffix Vectors
3. Computing maximal repeats
4. Conclusion

# Motivation

Detecting repeats in long biological sequences.

Adapted index structure.

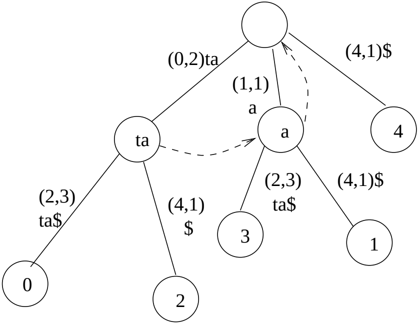
## Notations

$y$ is a sequence of length $n$ on the alphabet $A$.
\$ is a terminator symbol.

## Suffix tree

- index structure;
- all substrings represented;
- edges labeled (begin position, length);
- leaves represent suffixes.

Suffix tree of tata$



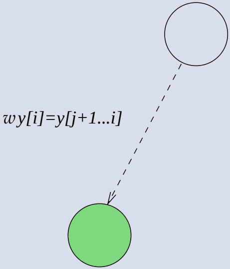
# Ukkonen's algorithm

- On-line algorithm
- Construction split into $n$ phases which are also split into extensions.
- During the phase $i$, construction of the implicit tree of $y[0..i]$ from the one of $y[0..i-1]$.
- During the extension $j$ of the phase $i$, the suffix $y[j+1..i]$ is added to the tree.
- The last added substring is $w = y[j+1..i-1]$.

# The 3 rules

Ukkonen's algorithm is based on 3 rules expressed by Gusfield:

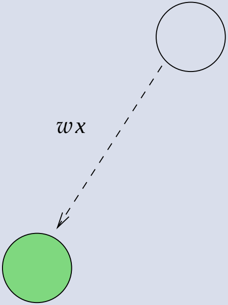
## Rule 1



# The 3 rules

Ukkonen's algorithm is based on 3 rules expressed by Gusfield:

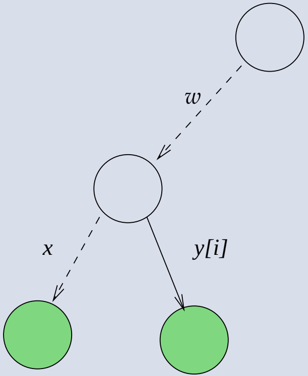
## Rule 2



# The 3 rules

Ukkonen's algorithm is based on 3 rules expressed by Gusfield:

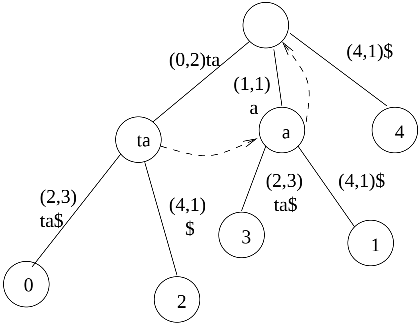
**Rule 2**

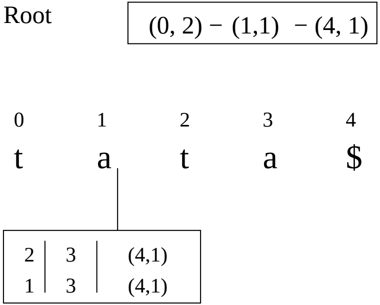


# Some properties

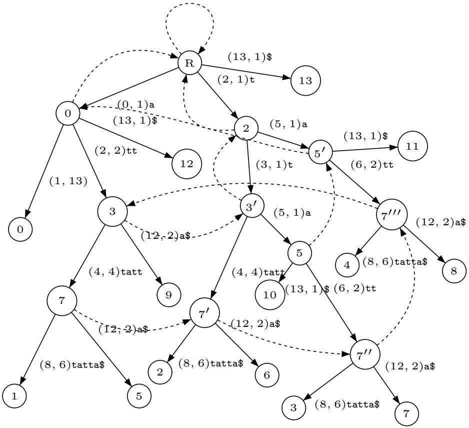
- leaves are added in increasing order;
- rule 1 does not need any treatment;
- phase $i$ begins at the extension $j_\ell + 1$, where $j_\ell$ is the number of the last created leaf;
- phase $i$ ends at the first extension $j > j_\ell$ such that rule 3 is applied.

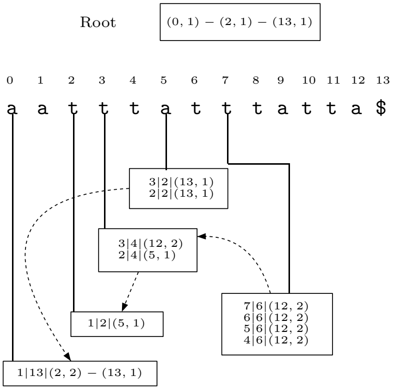
# Introduction to suffix vectors



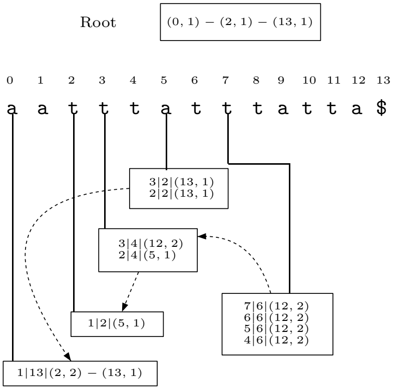
# Introduction to suffix vectors



# Introduction to suffix vectors

- Alternative data structure to suffix trees
- same information in reduced space
- introduced by K. Monostori in 2001



# Introduction to suffix vectors

## Definition

A succession of boxes whose lines contain:

- the depth of the node;
- the natural edge;
- the edge list.

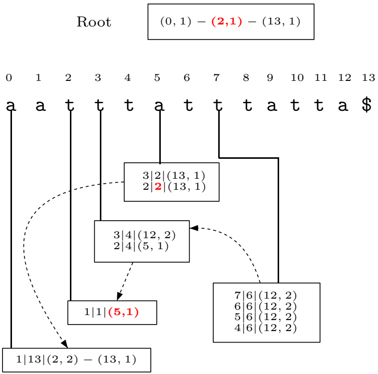
The root is a special box.

## Notations

# Introduction to suffix vectors

## Example

**tatt** is a substring of $y$ ?
The root contains the edge $(2, 1)$ beginning by **t** leading to $B_2$.
The edge $(5, 1)$ by **a** leads to $B_5$.
The natural edge begins by **tt**.



# Compact a vector

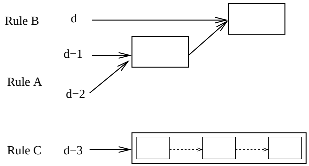
**Definition**

A *group of nodes* is a set of nodes which are in the same box and have exactly the same edges.

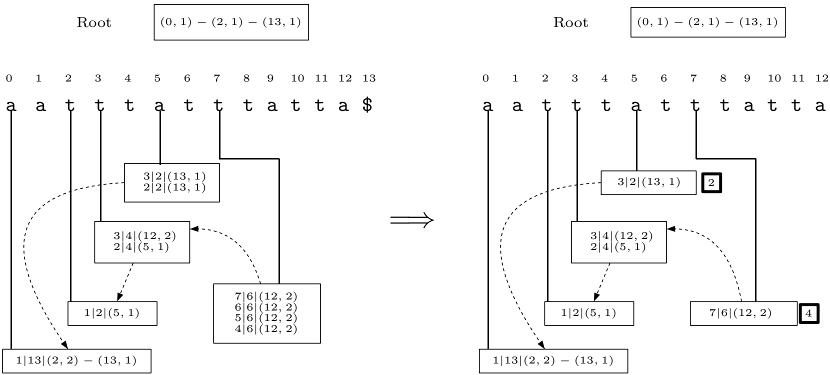
## Compact suffix vectors

3 rules of compaction of a box:

- **Rule A** the node with depth $d-2$ has the same edges as the node with depth $d-1$,
- **Rule B** the node with depth $d-1$ has the same edges as the node with depth $d$ and some extra edges,
- **Rule C** the node with depth $d-3$ has different edges to the node with depth $d-2$.



# Compacting $\mathcal{V}$(aatttatttatta$)



$$y \xrightarrow[O(n)]{\text{Monostori}} \text{Extended vector} \xrightarrow[O(n)]{\text{Monostori}} \text{Compact vector}$$

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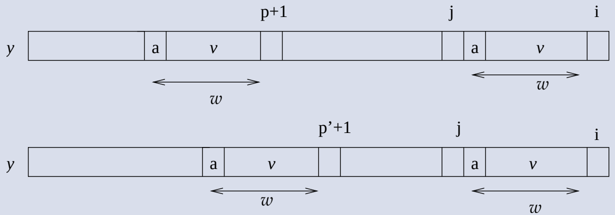
# On-line construction of a compact vector

**Proposition**

When an edge is added to the node $w$ of depth $d$ in a box $B_p$, this edge will be added to all the nodes in $B_p$ of depth smaller then $d$ in the group of nodes of $w$.



# On-line construction of a compact vector

Skip $k-1$ extensions where $k$ is the number of the nodes in the group into the edge is added.

1. Introduction
   - Motivation
   - Suffix trees
   - Ukkonen's algorithm
2. Suffix Vectors
   - Introduction
   - Compact Suffix Vectors
   - On-line construction of a compact suffix vector
3. **Computing maximal repeats**
4. Conclusion

### Definition

A maximal repeat in a string is a substring such that there exist at least 2 occurrences : $a_1ub_1$ and $a_2ub_2$ with $a_1 \neq a_2$, $b_1 \neq b_2$ and $a_1, a_2, b_1, b_2 \in A$.

### Example

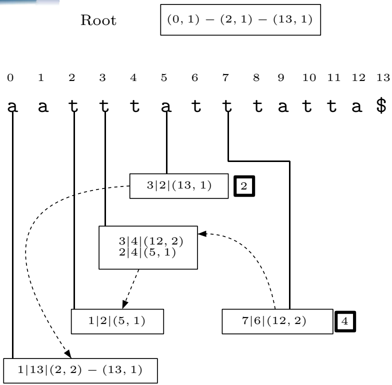
$y =$aatttatttatta$

tta is a maximal repeat at positions 5 and 12.

# Applying to suffix vectors

**Proposition**

The deepest node of each group of nodes represents a maximal repeat.




## Example

Boxes 0, 2, 5 et 7 are reduced:
`a`, `t`, `tta`, `atttatt` are maximal repeats.
Box $B_3$ is extended, the 2 lines have different edges:
`att`, `tt` are maximal repeats.

# Conclusion

More economical construction of the compact suffix vector.

Linear method to compute maximal repeats with a compact suffix vector.